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Gaussian Plume Model, Maximum Ground Concentration and Fumigation for Continuous Sources

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Abstract

THE crosswind and normal of dispersion modeling are obtained to obtain the concentration using Gaussian plume model. There are different shapes of the standard deviations such as power law and Briggs formula for estimating concentration of the pollutants. In this work power law and Briggs formula are used to obtain the Gaussian concentration of plume model, maximum concentration at the surface and the fumigation formula. Three predicted models are comparing to the measured concentration of Iodine-135 in unstable condition. One obtains that the Gaussian concentration in centerline with BNL is the best with the measured concentration than Gaussian concentration using Briggs formula which is in good result. Also, the statistical techniques appear that the data is located inside a factor of two. Also, the maximum Gaussian concentration with the Briggs and B N L are larger values the measured concentration of I^{135} and is located inside a factor of four.

Keywords: Dispersion Coefficients; Power Law and Briggs Formula; Maximum Gaussian Concentration.

Introduction

The Gaussian plume model is the most widely used for obtaining airborne radionuclide exposure within 80 km of the release point. This model is widely used because (a) It produces results similar to any other model when comparisons are made between predictions and experimental data, (b) mathematical operations are easily performed, (c) it is appealing conceptually, (d) it is suitable to the random nature of turbulence, (e) it is solved of the Fickian diffusion equation.

This maximum ground concentration (MGC) occurs through the centerline(y=0) at the surface (z=0) and at distance of maximum concentration depending on the explicit ways in which σ_y andσ_z increase with distance, x, [1].

Hanna et al., [2], and Sharan et al. [3] used the modified formulas in calculating the dispersion parameters under law wind speed in stable conditions. The effective of eddy diffusivity on the mimics of behaviour of diffusion equation was investigated by Essa et al. [4] and Essa et al. [5] obtained the solved of advection-diffusion equation in third dimensions using Hankel transform. Essa et al. [6] investigated the effect of wind speed which consists of power and logarithmic laws to obtain the solution of diffusion equation.

In this paper, the Gaussian formula is used in third dimensions for plume model, maximum concentration and fumigation concentration at the surface. Power law and Briggs formula [7] are used to obtain the dispersion parameters. Comparing the Gaussian formula model, maximum concentration at surface, the fumigation formula concentration and observed of Iodine-135 in unstable condition.

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Description Formulations

The Gaussian model is discussed because it is still the basic workhorse for dispersion calculations and it gives the concentration results which good with observed concentration data using constants values of dispersion parameters and wind speed. The origin of the Gaussian model is found in work by refs. [8-12]. Consider a continuous source of strength Q (g/s or Bq/s) at effective height "H" above the ground. Let that the wind velocity "u" is uniform, the concentration C (g/m^3, Bq/m^3) is obtained by the formula:

$$C(x,y,z) = \frac{Q}{2\pi u\sigma_y\sigma_z} e^{-\frac{y^2}{2\sigma_y^2}} \left[e^{\frac{-(z-H)^2}{2\sigma_z^2}} + e^{\frac{-(z+H)^2}{2\sigma_z^2}} \right] \quad (1)$$

the coordinate "y" refers to horizontal direction at right angles to the axis with y equals zero at center line and "z" is the normal height above the ground. σ_y *and* σ_z are standard deviations which depend on horizontal distance "x" and stability. $H = h_s + \frac{3w_0}{u} D$, where,"$h_s$" is the stack height, "$w_o$" is the initial plume speed and "D" is the stack diameter.

McElroy and Pooler's [13] diffusion experiment in St. Louis was used by Briggs [7] to develop the formulas given in Table 1.

where, A is extremely unstable, B is modularity unstable, C is slightly unstable, D is neutral condition and E , F are slightly and modularity stable.

Smith [14] summarized the Brookhaven National Laboratory (BNL) formulas which are based on hourly average measurements out to about 10 Km of diffusion of no buoyant plume release from height 108m.

$$\sigma_y = ax^b \quad and \quad \sigma_z = cx^d \quad (2)$$

Where, the constant values of the parameters a, b, c and d are given in Table 2.

Maximum Ground Concentration and Fumigation

Differentiate Eq. (1) with respect to "x" and set the result equal to zero to determine the maximum concentration at maximum downwind distance when $\sigma_y \propto \sigma_z$, this occurs at the distance downwind where $2\sigma_z^2 = H^2$, where "H" is the effective height $h_s + \Delta h$. The maximum concentration at that distance is obtained by the formula:

$$C_{max} = \frac{2Q}{\pi H^2 eu} \frac{\sigma_z}{\sigma_y} \quad (3)$$

Experience gives that the critical distance is a few tens of stack height (h_s) downwind. There is a critical wind speed at which C_{max} itself is a maximum if there is any plume rise at all. This phenomenon is called "high wind fumigation" and can persist for hours. The term "fumigation" context means a situation in which high concentrations are brought to the ground from an elevated plume. The critical wind speed (u_c) and maximum concentration at that speed.

"Limited mixing fumigation" occurs when diffusion or penetration of the plume restricted by an inversion. If the inversion height is "H" and it is about 500m. If the inversion height is "H" and let that the vertical distribution of the plume is uniform from the ground to "H", then the concentration at the surface is obtained by:

$$C_{fum} = \frac{Q}{\sqrt{2\pi} uH\sigma_y} \quad (4)$$

Tennessee Valley Authority experience shows that limited mixing fumigation more frequently gives the highest ground concentrations at their very tall stacks ($h_s > 100m$), whereas high wind fumigation is more frequently critical at their shorter stacks.

This can be seen as follows: consider the ratio of the predicted concentrations of the two methods

$$\frac{C_{LM}}{C_{HW}} = \left(\frac{\pi}{2}\right)^{0.5} \frac{H^2 e}{2z_i\sigma_z} \quad (5)$$

If one assumes that $H^2 = 2\sigma_z^2$at the maximum point, then Eq. (5) becomes:

$$\frac{C_{LM}}{C_{HW}} = 2.4\frac{H}{z_i} \qquad (6)$$

If mixing height (z_i) is 500m, then Limiting mixing fumigation will be more important than high wind fumigation when the effective plume height is larger than about 200m.

The wind speed “u” appearing in the basic Gaussian plume formula, Eq. (1) should be average value over the plume depth is generally recognized. The wind velocity must be estimated by using observations near the surface as follows:

One uses the power law as follows:

$$u = u_{10}\left(\frac{z}{10}\right)^p \qquad (7)$$

Where z is height in meter, u_{10} is the values of wind velocity at reference height at 10m, this formula is used by several of the U.S. Environmental Protection Agency (EPA) models with values of the parameter P estimated by Irwin (1979b)which given in Table 3.

Results and Discussions

Measured concentration data of I^{135} isotope concentration was obtained from dispersion as experiments conducted in unstable condition air samples which were collecting around the Egyptian Atomic Energy Authority (EAEA) at vertical height equals 0.7 m above surface with a stack height equals 43m, for twenty-four hours working. Each air samples are collected at half hour with roughness length “z_0” equals0.6 cm.The meteorological data are taken from Essa and El-Otaify [15] in Table 4. The measured concentration, Gaussian and maximum Gaussian concentrations by Eqs. (1), and (3) below the plume center lines of Iodine-135 (I^{135}) isotope using the dispersion parameters σ_y and σ_z in Briggs and BNL are shown in Table 5. The comparing between Gaussian, maximum Gaussian concentrations and measured concentrations of radioactive I^{135}via horizontal distance “x” is shown in Fig. 1. in unstable condition. Fig. 2. explains the relation between the two proposed and measured concentrations.

From the two figures, one finds that Gaussian concentrations using power law of dispersion parameters (BNL) are the best values with the measured concentration of I^{135} than the Gaussian concentration using Briggs formulas for dispersion parameters but the two Gaussian concentrations are located inside a factor of two. Also, some points of maximum Gaussian concentrations using Briggs and BNL are located inside a factor of two and others located inside a factor of four.

Statistical Technique

Comparing between Gaussian, maximum Gaussian and measured concentrations is introduced by [16].

NMSE is the Normalized Mean Square Error, FB is the Fraction Bias, COR is the Correlation coefficient and FAC2 is the Factor of Two.

From Table 6. the Gaussian concentration at center line using Briggs and BNL of dispersion parameters achieved 100% and 0.94% with measured concentration data respectively. Also, the statistical shows that the Gaussian concentration using BNL of dispersion parameters is the best for NMSE, FB, and COR than the Gaussian concentration using Briggs of dispersion parameters. Also, this work shows that the maximum Gaussian concentration using Briggs and BNL of dispersion parameters inside a factor of four and other statistical parameters are not good because of the large values of the concentration.

Conclusions

In this work, one gets the Gaussian, maximum Gaussian and fumigation concentrations using Briggs and BNL of standard parameters in “y” and “z” directions and comparing the Gaussian, maximum Gaussian and observed concentration of I^{135}.

One obtains that the Gaussian concentration in center line with BNL is the best with the measured concentration than Gaussian concentration using Briggs formula which is in good result. Also, the statistical techniques appear that the data is located inside a factor of two. Also, the maximum Gaussian concentration with the Briggs and BNL are larger values the measured concentration of I^{135} and is located inside a factor of four.

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Declaration of Conflict of Interest

The authors declare that there is no conflict of interest.

Table 1. Formulas recommended by Briggs (1973) for $\sigma_y(x)$ and $\sigma_z(x)$ 102<x<104m in urban area. A, B, C, D, E and F are extremely unstable, modularity unstable, slightly unstable, neutral condition, slightly stable and modularity stable respectively.

Stability	$\sigma_y(x)$	$\sigma_z(x)$
A-B	$0.32x(1+0.0004x)^{-1/2}$	$0.24x(1+0.001x)^{1/2}$
C	$0.22x(1+0.0004x)^{-1/2}$	$0.20x$
D	$0.16x(1+0.0004x)^{-1/2}$	$0.14x(1+0.0003x)^{-1/2}$
E-F	$0.11x(1+0.0004x)^{-1/2}$	$0.08x(1+0.00015x)^{-1/2}$

Table 2. Brookhaven National Laboratory (BNL) parameter values A, B, C, and D

Type	a	b	c	d
B	0.40	0.91	0.41	0.91
C	0.32	0.86	0.33	0.86
D	0.32	0.78	0.32	0.78
E	0.31	0.71	0.08	0.71

Table 3. The values of parameter (p) through stability conditions in urban area

A	B	C	D	E	F
0.15	0.15	0.20	0.25	0.40	0.60

Table 4. The meteorology data of 9 convective test runs at Inshas site from March to May 2006 [15].

Run numbers	Working hours of the source	Release rate (Bq)	Wind speed (m s^{-1})	Wind direction (deg)	W_* (ms^{-1})	P-G stability class	h (m)	Vertical distance (m)
1	48	1028571	4	301.1	2.27	A	600.85	5
2	49	1050000	4	278.7	3.05	A	801.13	10
3	1.5	42857.14	6	190.2	1.61	B	973	5
4	22	471428.6	4	197.9	1.23	C	888	5
5	23	492857.1	4	181.5	0.958	A	921	2
6	24	514285.7	4	347.3	1.3	D	443	8.0
7	28	1007143	4	330.8	1.51	C	1271	7.5
8	48.7	1043571	4	187.6	1.64	C	1842	7.5
9	48.25	1033929	4	141.7	2.1	A	1642	5.0

Table 5. Observed, Gaussian and maximum Gaussian concentrations using Briggs and BNL of dispersion parameters for Run 9 experiments

Run no.	Downwind distance	Observed concentration	Gaussian Briggs	Gaussian Power Law	Maximum Gaussian Briggs	Maximum Gaussian Power Law
1	100	0.025	0.03536	0.01769	0.24964	0.32092
2	98	0.037	0.04555	0.02829	0.23267	0.29949
3	136	0.091	0.07356	0.06219	0.00745	0.00936
4	135	0.197	0.16703	0.20756	0.13712	0.15059
5	106	0.272	0.19282	0.28558	0.1349	0.17276
6	186	0.188	0.14353	0.20464	0.14745	0.14929
7	165	0.447	0.39462	0.42763	0.27482	0.30012
8	154	0.123	0.15769	0.13092	0.28417	0.31097
9	106	0.032	0.04808	0.03485	0.25191	0.3226

Table 6. The Comparison between Gaussian, maximum Gaussian using Briggs and BNL of dispersion parameters, and observed concentrations of I^{135} in unstable condition.

	NMSE	FB	COR	FAC2
Gaussian Briggs	0.07	0.12	0.98	1.05
Gaussian BNL	0.01	0.01	0.99	0.94
Maximum Gaussian Briggs	0.83	-0.20	0.03	3.24
Maximum Gaussian BNL	1.0	-0.36	-0.1	4.06

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نموذج عمود جاوسي ، أقصى تركيز أرضي والتبخير للمصادر المستمرة

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الملخص

الرياح المتقاطعة ونمذجة التشتت الطبيعية إستخدمت للحصول على التركيز باستخدام نموذج عمود جاوسي. هناك أشكال مختلفة للانحرافات المعيارية مثل قانون القوة وصيغة بريجز لتقدير تركيز الملوثات. في هذا العمل قانون القدرة وصيغة بريجز ، يتم استخدامها للحصول على التركيز الغوسي لنموذج العمود ، والتركيز الأقصى على السطح وصيغة التبخير. تقارن ثلاثة نماذج متوقعة بالتركيز المقاس لليود 135 في حالة غير مستقرة. يحصل المرء على أن تركيز جاوسي في خط الوسط مع معمل بروك هافن الوطني هو الأفضل مع التركيز المقاس من التركيز الغوسي باستخدام صيغة بريجز التي تكون في نتيجة جيدة. أيضا ، تظهر التقنيات الإحصائية أن البيانات تقع داخل عامل اثنين. أيضا ، فإن التركيز الغاوسي الأقصى مع برجز و معمل بروك هافن الوطني هي قيم أكبر للتركيز المقاس I^{135} وتقع داخل عامل أربعة.

الكلمات الدالة: معاملات التشتت، قانون القوة وصيغة بريجز، الحد الأقصى للتركيز الجاوسي من.