

## An Approach for Estimating the Deformation of Pulsar Included in Binary-System PSR B 1913+16

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**D**ISCOVERY of pulsars included in binary systems (BS) is dated to 1974, when Hulse and Taylor announced the parameters of pulsar PSR B 1913+16. It inspires great motivations for possible new physical measurements of the properties of neutron stars (NSs). We present a simple model to estimate the deformation in shape of two neutron stars bounded to each other in a binary system due to their mutual gravity. The model predicts the Bulk modulus of the pulsar – as an infinite nuclear matter. In particular, the model determines the stress and the strain of a pulsar included in PSR B 1913+16 binary system within the frame of Newtonian theory of gravity. The calculations show that, our approach is consistent with the model of two spherical neutron stars up to the eccentricity-squared. The model could evaluate the axial elongation of the observed NS as well as the eccentricity. The eccentricity is found to be of order  $10^{-5}$  and the elongation is of the order  $10^{-9}$  km.

**Keywords:** Infinite Nuclear Matter, Neutron star, Pulsar, Bulk modulus, Compressibility, Companion star.

### Introduction

Pulsars PSR B1913+16 (PSR J1915+1606) and PSR B 1534+12 would exhibit a rich set of potentially measurable relativistic effects. Subsequent studies have shown that the pulsar's companion must also be a neutron star [1, 2, 3]. The systems are essentially clean laboratories for testing relativistic gravity. The tidal and rotational stellar distortions are complicating factors for any dynamical analysis and the largest remaining complications depend on reference-frame accelerations related to the structure and dynamics of our Galaxy.

Deformation of neutron stars is controlled by the equation of state (EoS) of the neutron stars. The difficulty in estimating the neutron star deformation parameters is coming from the absence of physical models relating stress and strain on the matter of the neutron star. Using EoSs, several studies estimate the compressibility coefficients of the neutron stars to be in the range 180-240 MeV for finite heavy nuclei by performing effective interactions-microscopic calculations [4]. It is found that some effective Lagrangians having a bulk compression modulus in the range 280–350 MeV can predict correctly breathing mode energies in medium and heavy

nuclei [5]. A symmetric matter incompressibility parameter  $K_S = 240$  MeV fits the heavy-ion data [6]. Using a quantum hydrodynamic effective model in the mean-field approximation to the description of neutron stars, it could be predicted that, the compression modulus of nuclear matter is  $K = 257.2$  MeV [7].

It is possible to estimate the strain and stress on the surface of a pulsar included in a binary system in the frame of Newtonian dynamics. Horowitz and Kai Kadau's [8] published simulations of Coulomb solids for representing neutron star crust show that, the breaking strength of neutron star crust is about  $10^{10}$  times more than for terrestrial engineering materials such as metal-alloys where the strength is measured in fractions of a GPa, and the breaking strain is about 0.1. A stable and unique value of the infinite nuclear matter compression modulus ( $K_\infty = 288 \pm 20$  MeV) has been extracted using the masses of all known nuclei [9, 10]. The extraction method does not use any effective interaction or dynamic property like monopole resonance.

In the present work, the gravitational effect on the deformation of a pulsar included in a binary system will be modeled in order to calculate the potentials and the force components in the body

and on the surface of the pulsar. Section 1 will exhibit the basic model of a pulsar in a binary system. Section 2 shows the developed model to define the deformation of the pulsar using the terminology of ref. [11]. Section 3 are the results and discussions. Finally, section 4 is left for the conclusions.

#### *Basic Parameters of PSR B 1913+16*

The 1<sup>st</sup> binary pulsar system PSR B1913+16 was discovered by Hulse and Taylor in summer, 1974 [12] in a systematic search for new pulsars. The nominal pulse period of the pulsar is 59 ms. This short period was observed to be periodically shifted (Doppler Shift), which proves that the pulsar is a member of a binary system with an orbital period of 7.75 hours. With Kepler's third law and reasonable masses, one concludes from this that the system is rather narrow, having a diameter of roughly  $1 R_{\odot}$  correspondingly, the velocity of the pulsar  $\sim 10^{-3} c$  and it moves through a relatively strong gravitational field ( $GM/c^2 r \sim 10^{-6}$ ). These parameters show that several special and general relativistic effects should be observable. This calculations have been achieved over three decades with increasing accuracy [13]. The measurements on the system PSR B1913+16 yield enough information to determine all parameters of the system as shown in Table 1, and allow in addition a test of general relativity (GR)-results, provided that; the companion star has a negligible quadrupole moment and tidal interactions are small enough [14]. These conditions for a "clean" relativistic system are probably fulfilled.

#### *The model*

The proposed model aims to setup a calculation scheme of the deformation of a neutron star included in a binary system. The basic assumptions of the model are;

1. The deformation of the observed NS is due to the resultant forces of the Newtonian-gravitation.
2. The initial shape of NS is a sphere at apastron, and after the deformation at the periastron, the shape will be an ellipsoid of revolution with small eccentricity  $e$  about z-axis. The deformation is along the z-axis, preserves the radius in XY-plane, and occurs according to the classical tide-theory [15].
3. NS is, instantly, in thermodynamic equilibrium

(static NS,) and the mechanical balance is between the internal pressure force and the resultant gravitational force.

4. The spinning of NS about the z-axis will not affect the deformation of the NS, specifically for symmetric deformation.
5. The density of NS is uniform and denoted by  $\rho_{NS}$ .

The proposed model of the pulsar and a companion NS included in binary system and orbiting about their common center of mass is illustrated in Fig. 1.

The effect of the companion (NS) on deformation of the observed pulsar is shown in Fig. 2. The pulsar orbital motion includes an acceleration toward the companion (NS). This acceleration affects any surface element on pulsar, as it orbits with the companion NS. The resulting deformation corresponds to the pull at the pulsar's center. Any surface elements on the companion (NS) side of the pulsar (point B) is pulled by the companion (NS) with a force that is slightly greater than it would be at the center. The surface of the pulsar which is nearest side to the companion (NS) behaves as if they suffer a slight additional attraction toward the companion (NS). On the other side the surface on the far side from the companion (NS) (point A) are pulled by the companion (NS) with a force that is slightly weaker than it would be at the center. Also it is clear from the Fig. 2, that, at the points P and Q there is weaker deformation effects than at points A and B because of small compressibility of Neutron star matter. It is possible to reason why must the deformations of the nearest and the farthest parts of the pulsar be symmetric? The answer is that, the acceleration  $a$  of an element  $\Delta m$  on the pulsar surface is evaluated from Newton's laws followed by Taylor-expansion for the variable ( $d = d_0 \mp R$ ) up to order one about  $d_0$  (the z-coordinates of the center of mass), we get

$$a \approx -\frac{GM}{d_0^2} \pm \frac{GM}{d_0^3} R = -\frac{GM}{(d_0^2)(d_0 \mp R)}.$$

That is, at any instance of observation time the relative velocities of the surface elements cause a type of symmetric instantaneous deformations of the whole body of the pulsar. The instantaneous deformations reshape the spherical NS to an ellipsoid of revolution with the major axis is along the line passing through the centers of the two NSs.

**TABLE 1.** The classical orbital elements, astrometric parameters and post Keplerian parameters for the most famous relativistic binary system PSR B1913+16 taken from reference [14].

	<b>Parameters</b>	
Classical orbital elements	Orbital period $p_b$ [d]	0.322997462
	Eccentricity	0.6171308(4)
	Semi major axis $a_p \sin i/c$ [s]	2.3417592(19)
	Periastron length $\omega_0$ [deg]	226.57528(6)
	Periastron passage $T_0$ [MJD]	46443.99588319(3)
Astrometric parameters	Spin period $P$ [ms]	59.029997929613(7)
	Braking rate $\dot{P}$ [ $10^{-18}$ ]	8.62713(8)
	Right ascension (J2000)	19: 15: 28.0002
	Declination (J2000)	16: 06: 27.4043
	Dispersion [ $pc\ cm^{-3}$ ]	168.770
	Timing accuracy [ $\mu s$ ]	15
Post Keplerian parameters	Periastron shift $\dot{\omega}$ [deg/yr]	4.226621(11)
	Grav/Doppler effect $\gamma RD$ [ms]	4.295(2)
	Shapiro time delay $r$ [ $\mu s$ ]	—
	Orbital inclination $s = \sin i$	—
	Orbit decay $\dot{P}_b$ [ $10^{-12}$ s/s]	-2.422(6)

The gravitational potential  $U(\mathbf{r})$  is calculated analytically inside and outside an ellipsoid and sphere by integration over the mass-element

$dm' = \rho_{NS} dV'$  of a single NS using Newton's law of gravitation potential.

$$U(\mathbf{r}) = -G \int_{V_{NS}} \frac{\rho_{NS}}{|\vec{r} - \vec{r}'|} dV', \quad dV' = \rho' d\rho' d\phi' dz' \quad (1)$$

Where

$$\rho' \in [0, f_s(z')], \quad f_s(z') = \begin{cases} +R \sqrt{1 - (z'/R)^2} & \text{for a sphere,} \\ +R \sqrt{1 - (z'/\tilde{R})^2} & \text{for an ellipsoide} \end{cases}$$

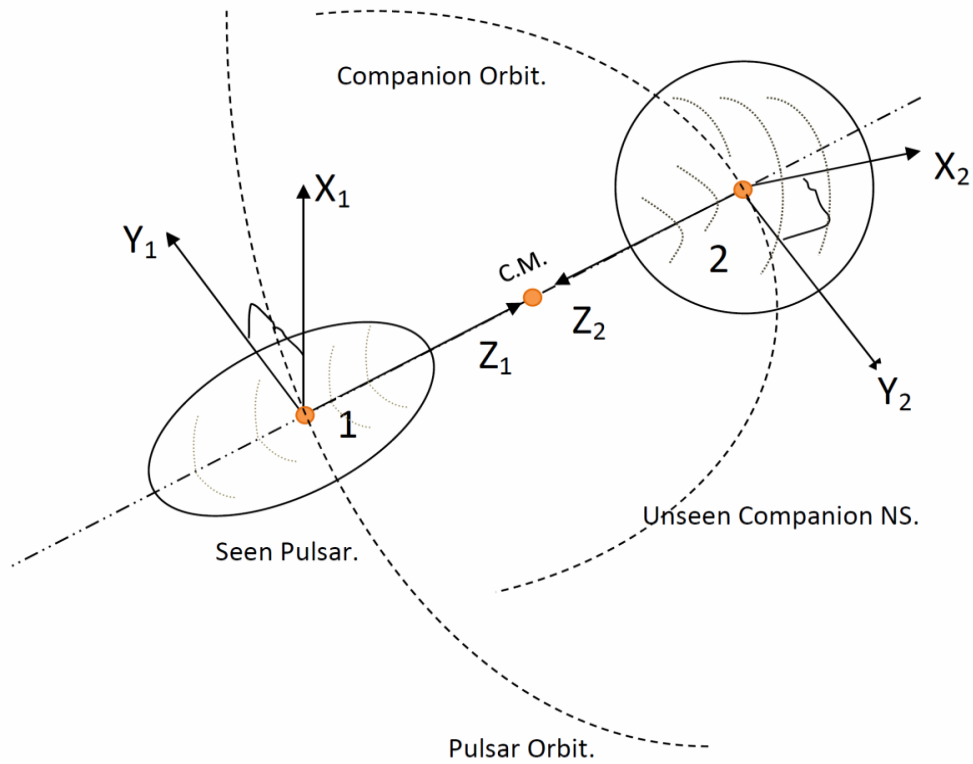


Fig. 1. Illustration by the authors of the binary system of two NSs are orbiting each other about their common center of mass (C. M.) The observed pulsar is deformed to an ellipsoid (= a body of revolution). The illustration represents the coordinate system of each NS.

$$\rho' \in [0, f_s(z')], \quad f_s(z') = \begin{cases} +R \sqrt{1 - (z'/R)^2} & \text{for a sphere,} \\ +R \sqrt{1 - (z'/\tilde{R})^2} & \text{for an ellipsoid,} \end{cases}$$

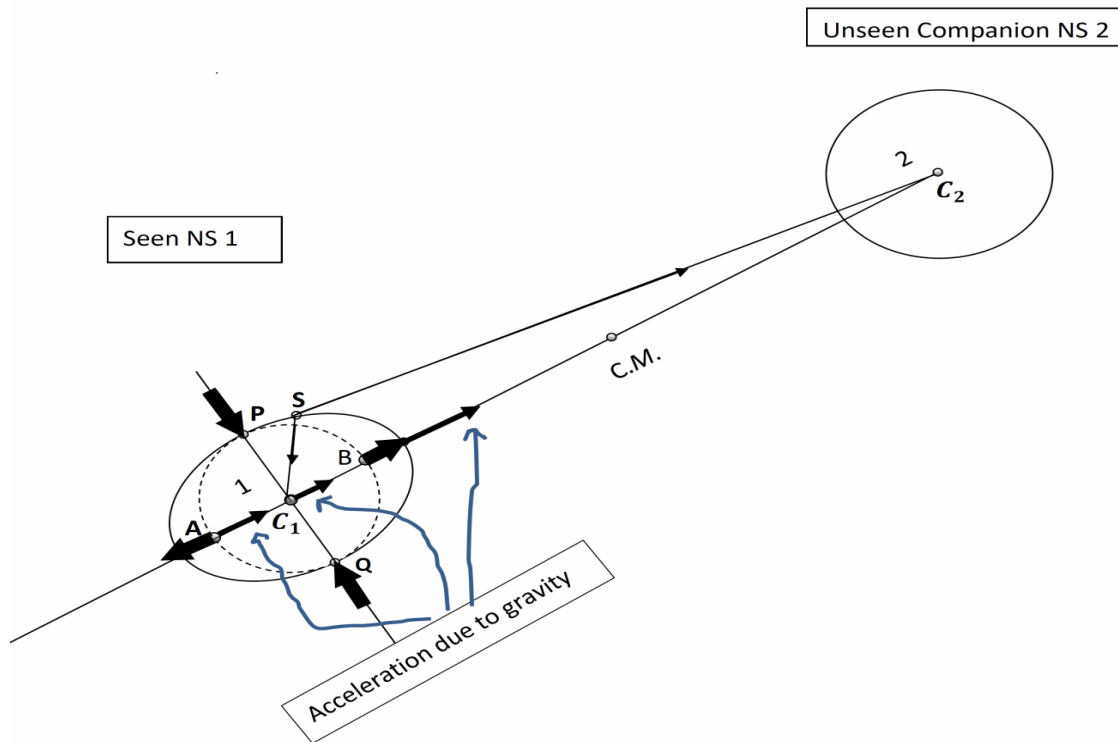
$$\tilde{R} = \frac{R}{\sqrt{1 - e^2}}, \text{ for the spherical shape.} \tag{2a}$$

$$|\vec{r} - \vec{r}'| = \sqrt{(\rho \cos\phi - \rho' \cos\phi')^2 + (\rho \sin\phi - \rho' \sin\phi')^2 + (z - z')^2}$$

$$= \sqrt{\rho^2 + \rho'^2 - 2\rho\rho' \cos(\phi - \phi') + (z - z')^2}.$$

Along the z-axis .

$$|\vec{r} - \vec{r}'| = \sqrt{\rho'^2 + (z - z')^2}. \tag{2b}$$



**Fig. 2.** Illustration by the authors for the binary system, where the observed NS (the pulsar) is deformed according to the tide-theory [15]. The heavy arrows represent the instantaneous symmetric deformations due to different accelerations of the body-elements as seen in the extrinsic coordinates of the pulsar due to the companion NS.

Performing the integration procedure for  $\rho' d\rho'$ , and then obtain the series expansion up to order  $e^2$ . The integration for  $(\phi')$  is cyclic due to the assumed geometrical symmetry. The last integration for  $(z')$  is done analytically

for each series term up to order  $e^2$ . Finally the potentials inside ( $|z| \leq \bar{R}$ ) and outside ( $|z| > \bar{R}$ ) the ellipsoid (Ei, Eo) and the sphere (Si, So) could be written as;

$$U^{Ei}(z; R, e) = \frac{2}{3} \pi G \rho_{NS} (-3R^2 + z^2) - e^2 \frac{2}{15} \pi G \rho_{NS} (5R^2 + 2z^2) + o(e^4) \tag{3}$$

$$U^{Eo}(z; R, e) = -\frac{4\pi G \rho_{NS} R^3}{3z} - e^2 \frac{2\pi G \rho_{NS} (2R^5 + 5R^3 z^2)}{15z^3} + o(e^4) \tag{4}$$

$$U^{Si}(z; R) = \frac{2}{3} \pi G \rho_{NS} (-3R^2 + z^2), \tag{5}$$

$$U^{So}(z; R) = -\frac{4\pi G \rho_{NS} R^3}{3z} \tag{6}$$

The integrant in equation (1) includes singularity at for the case  $|z| \leq \tilde{R}$ . This singularity point requires mathematical labor for performing the integration over  $z'$ .

To find the stress, first obtain the negative of the gradient of the Gaussian potential inside (i) and outside (o) the ellipsoid (E) and the sphere (S) to get the z-component of the forces per unit mass:  $\mathbf{F}_z^{\text{Ei}}(z)$ ,  $\mathbf{F}_z^{\text{Eo}}(z)$ ,  $\mathbf{F}_z^{\text{Si}}(z)$ , and  $\mathbf{F}_z^{\text{So}}(z)$  respectively. They are calculated analytically and found to be:

$$\mathbf{F}_z^{\text{Ei}}(z) = \frac{4}{15} \pi \mathbf{G} \rho_{\text{NS}} (-5 + 2e^2) z. \quad (7)$$

$$\mathbf{F}_z^{\text{Eo}}(z) = -\frac{2\pi \mathbf{G} \rho_{\text{NS}} \pi R^3 (6e^2 R^2 + 5(2 + e^2)z^2)}{15z^4}. \quad (8)$$

$$\mathbf{F}_z^{\text{Si}}(z) = -\frac{4}{3} \pi \mathbf{G} \rho_{\text{NS}} z. \quad (9)$$

$$\mathbf{F}_z^{\text{So}}(z) = -\frac{4\pi \mathbf{G} \rho_{\text{NS}} R^3}{3z^2}. \quad (10)$$

Where  $R$  is radius,  $G$  is gravitation constant,  $\rho_{\text{NS}}$  is density, and  $e$  is eccentricity.

The z- component of the resultant of the forces per unit mass  $\mathbf{F}_z^{\text{Res}}$  on the mass element  $dm = dA_{\perp} dz \rho_{\text{NS}}(z)$  at the surface of the observed NS is found by:

$$\mathbf{F}_z^{\text{ERes}} = \mathbf{F}_z^{\text{Ei}}(\tilde{R}_1) - \mathbf{F}_z^{\text{Eo}}(d_{\text{peria}} - \tilde{R}_1) \quad (11)$$

$$\mathbf{F}_z^{\text{SRes}} = \mathbf{F}_z^{\text{Si}}(R_1) - \mathbf{F}_z^{\text{So}}(d_{\text{apas}} - R_1) \quad (12)$$

Where  $d_{\text{peria}}$  and  $d_{\text{apas}}$  are the distances between the centers of the two neutron stars at periastron and apastron, respectively.  $R_1$  is the radius of the observed NS, when it was a sphere.

$$\tilde{R}_1 = \frac{R_1}{\sqrt{1 - e^2}} \approx R_1 \left( 1 + \frac{1}{2} e^2 \right). \quad (13)$$

Then the stress ( $\sigma_{zz}$ ) can be written as,

$$\sigma_{zz} = \frac{dF_z}{dA_{\perp}} = \frac{(\mathbf{F}_z^{\text{ERes}} - \mathbf{F}_z^{\text{SRes}}) dm}{dA_{\perp}} = (\mathbf{F}_z^{\text{ERes}} - \mathbf{F}_z^{\text{SRes}}) \rho_{\text{NS}} \Delta z$$

$$\sigma_{zz} = \frac{dF_z}{dA_{\perp}} = \frac{(\mathbf{F}_z^{\text{ERes}} - \mathbf{F}_z^{\text{SRes}}) dm}{dA_{\perp}} = (\mathbf{F}_z^{\text{ERes}} - \mathbf{F}_z^{\text{SRes}}) \rho_{\text{NS}} \Delta z \quad (14)$$

The volume of ellipsoid and sphere are given respectively by,

$$\mathbf{V}_{\text{sphere}} = \frac{4}{3} \pi R_1^3 \mathbf{V}_{\text{sphere}} = \frac{4}{3} \pi R_1^3, \quad (15)$$

The strain given by,

$$\epsilon = \frac{dV}{V} = \left( \frac{\mathbf{V}_{\text{Ellipsoid}} - \mathbf{V}_{\text{sphere}}}{\mathbf{V}_{\text{sphere}}} \right) = \frac{\tilde{R}_1 - R_1}{R_1} \approx \frac{1}{2} e^2 \quad (16)$$

The elastic modulus of volume-deformation is known as the bulk modul and is defined by (BM)  $= \frac{\text{stress}}{\text{strain}}$

The compressibility is a thermodynamic coefficient, defined by:

$$\therefore K = \frac{1}{BM}$$

Then the compressibility can be written as,

$$K = \frac{\text{strain}}{\text{stress}} = \frac{(\tilde{R}_1 - R_1)}{R_1 (\mathbf{F}_z^{\text{ERes}} - \mathbf{F}_z^{\text{SRes}}) \rho_{\text{NS}} \Delta z} \quad (17)$$

That is the compressibility is a function of the NS-parameters ( $R, \rho_{\text{NS}}, e$ ). The published estimations of the compressibility are coming from the proposed equations of state of the infinite nuclear matter [16,17]. Our model is succeeded in estimating the value of the deformation along z-axis and give an acceptable picture of the effect of the gravitational interaction on the infinite nuclear matter. In future work the authors will introduce the modification to the model, which enable the astrophysicists to estimate the deformation magnitude of a NS in BS.

## Results and Discussions

The authors find Newtonian potentials and forces (see equations 4-6 for calculating potentials and equations 7-10 for calculating z-components of the forces) according to the proposed model for the observed pulsar as a

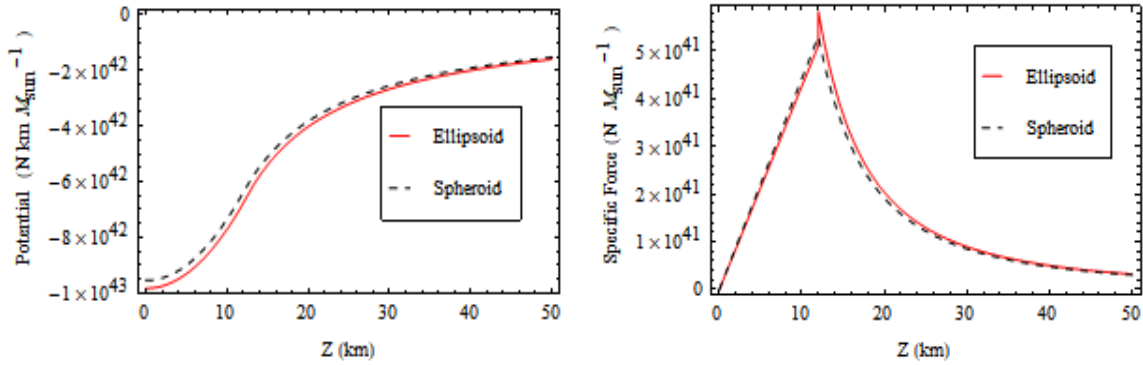


Fig. 3. The calculated gravitational potentials (a) and forces (b) for a single compact object (pulsar) as a function of the distance along z-axis. The dashed lines are the usual known Gaussian potentials and forces inside and outside spherical body ( $R_1=12$  km). The solid lines are the same calculation for ellipsoidal body ( $R_1=12$  km, and  $e=0.3$ ).

The potentials in Fig. 3a have a characteristic and constant values at  $Z = 0$ . That are,

$$U^{Ei}(0; R_1, e) = -2\pi G \rho_{NS} R_1^2 - e^2 \frac{2}{3} \pi G \rho_{NS} R_1^2 + o(e^4)$$

, and  $U^{Si}(0; R_1) = -2\pi G \rho_{NS} R_1^2$ . The

potential at the surface of neutron star ( $Z = R_1$ ) is another characteristic quantity. That are,

$$U^{Ei}(\tilde{R}_1; R_1, e) = -\frac{4}{3} \pi R_1^2 G \rho_{NS} - e^2 \frac{8}{5} \pi R_1^2 G \rho_{NS} + o(e^4)$$

, and  $U^{Si}(R_1; R_1) = -\frac{4}{3} \pi R_1^2 G \rho_{NS}$ . The notch

at  $12$  km  $12$  km in figure 3b is due to the

approximation  $\tilde{R}_1 \approx R_1 (1 + 1/2 e)$  through

out the numerical calculation. The discontinuity in the curves of the force is due to the different functions describe the inside and the outside potentials which must coincide at the surface of the NS. The potential and the force of the ellipsoidal body resulting from the infinitesimal deformation of the spherical body are consistent and have similar characteristics and behaviors as in the case of spherical bodies.

The compressibility ( $K$ ) is calculated for the proposed model according to equation (17) and shown in Fig. 4.

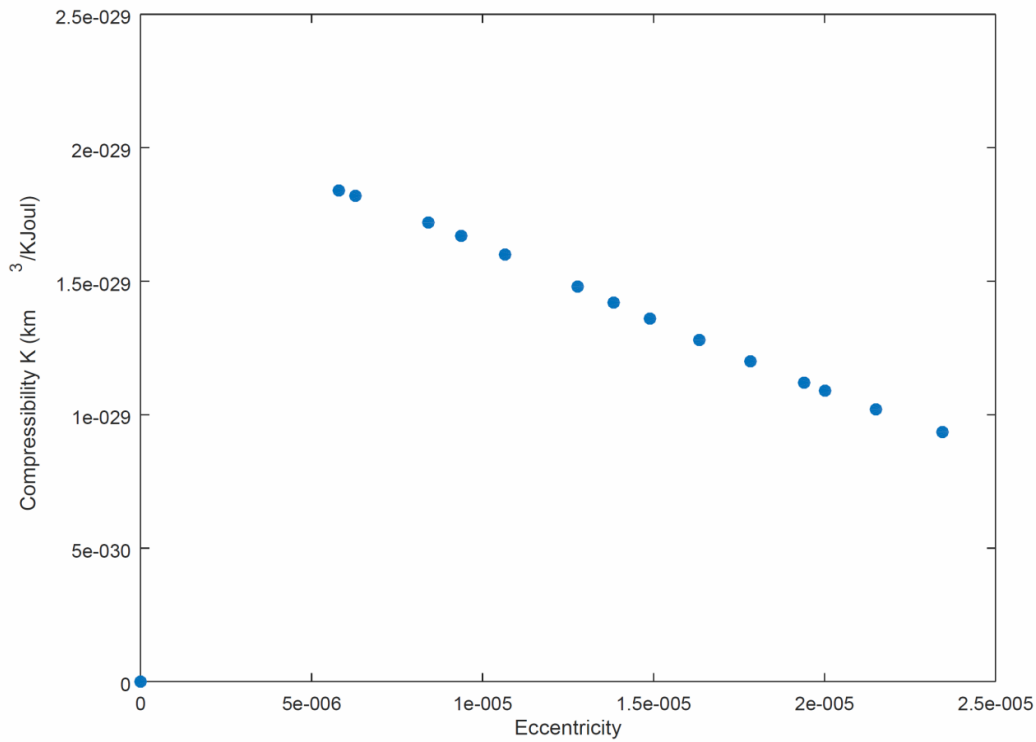
compact object included in a binary system. Figure 3a presents the comparison between the spherical and ellipsoidal single pulsar potentials and Figure 3b is a comparison between forces, where the eccentricity and the radius of the pulsar are assumed to be  $e = 0.3$ ,  $R_1 = 12$  km.

It is clear that, as the deformation increases the eccentricity increases and the compressibility decreases, i.e. the same gravitational forces could cause more deformation. That is the neutron star material becomes more soft.

The compressibility of finite nuclear matter in its ground state is calculated by using the highly symmetric skyrme potential of a shell model in the paper of M. M. Sharma [19]. The conclusion is that, the compressibility  $K$  should be between  $270 - 300$  MeV for the neutron-pair separation energy to be consistent with the experimental data on the charge - radii.

M. M. Sharma had taken the saturation density to be in the range  $0.160$  to  $0.170$   $f m^{-3}$ , while the saturation binding energy is fixed at  $-16.0$  MeV and the effective mass is from  $0.79$ . There is a strong correlation between the neutron-pair separation energy and the compressibility  $K$ .

The infinite nuclear matter could be studied through its equation of state. K. Hassaneen and H. Mansour in reference [18] had calculated the binding energy per nucleon (E/A) using a self-consistent Greens function method of the Brueckner-Hartree-Fock approach. Several values of the compressibility  $K$  of infinite



**Fig. 4.** The compressibility of the neutron star calculated based on the proposed model at different eccentricities

nuclear matter are obtained for different potential models at effective masses **0.566 to 0.618** and saturation densities from **0.160 to 0.374 fm<sup>-3</sup>**. The compressibility *K* is found to be in the range **203 to 337 MeV**.

The suggested model could predict each of the *BM, e, Δz* of the deformed pulsar. Based on the assumptions of the model, table 2 takes the predictions of the compressibility *K* (in

MeV), based on calculated values from M. M. Sharma [19] and K. Hassaneen and H. Mansour in reference [18], and calculates the bulk modulus (*BM*), the eccentricity (*e*), and the elongation along z-axis (*Δz*). The conversions of units from *MeV* to *kJoule* for *K*, requires the value of saturation nuclear density. It is taken to be **ρ<sub>0</sub> = 0.178 fm<sup>-3</sup>**, which is the average values of *ρ<sub>0</sub>* used in [18, 19]. The conversion equation for is:

$$K \left( \frac{km^3}{kJoul} \right) = \left( \frac{K(MeV) \times \rho_0(fm^{-3}) \times 10^{-9} \times F_1}{F_2} \right)^{-1} \frac{km^3}{kJoul},$$

$$F_1 = 7645373 \text{ MeV}^4 \text{ V} (MeV \text{ V} / fm^3),$$

$$F_2 = 5.00 \times 10^{-33} \text{ GeV}^{-4} / (km^3 / kJoul) \tag{18}$$

The behavior, from Table 2, of the eccentricity and *Δz* are monotonic decreasing with the compressibility *K* of the nuclear matter in the pulsar, which shows the stiffness of the NS materials and put a restriction on the acceptable

equations of states. It appears that, the estimated deformation of the pulsar due to its companion has the order of (**~10<sup>-5</sup>**) for eccentricity, and the mean elongation along z-axis is of the order **10<sup>-9</sup> km**. These small orders support the model assumptions.



**TABLE 2.** The values of the compressibilities(K) of the neutron star material (infinite nuclear matter,) The Bulk Modulus (BM), the eccentricity (e), and the elongation along the z-axis . The 1<sup>st</sup> column is taken from reference [18,19].

$K$ (MeV)	$K$ ( $km^3/KJoul$ )	$BM$ ( $N/km^2$ )	Eccentricity $e$	$\Delta z$ (km)
200	$1.84 \times 10^{-29}$	$1.087 \times 10^{29}$	$5.798 \times 10^{-6}$	$4.034 \times 10^{-10}$
203	$1.82 \times 10^{-29}$	$1.099 \times 10^{29}$	$6.283 \times 10^{-6}$	$4.737 \times 10^{-10}$
214	$1.72 \times 10^{-29}$	$1.163 \times 10^{29}$	$8.415 \times 10^{-6}$	$8.498 \times 10^{-10}$
220	$1.67 \times 10^{-29}$	$1.198 \times 10^{29}$	$9.375 \times 10^{-6}$	$1.055 \times 10^{-9}$
230	$1.60 \times 10^{-29}$	$1.25 \times 10^{29}$	$1.066 \times 10^{-5}$	$1.363 \times 10^{-9}$
249	$1.48 \times 10^{-29}$	$1.351 \times 10^{29}$	$1.278 \times 10^{-5}$	$1.960 \times 10^{-9}$
258	$1.42 \times 10^{-29}$	$1.408 \times 10^{29}$	$1.383 \times 10^{-5}$	$2.296 \times 10^{-9}$
270	$1.36 \times 10^{-29}$	$1.471 \times 10^{29}$	$1.489 \times 10^{-5}$	$2.662 \times 10^{-9}$
286	$1.28 \times 10^{-29}$	$1.563 \times 10^{29}$	$1.634 \times 10^{-5}$	$3.203 \times 10^{-9}$
305	$1.20 \times 10^{-29}$	$1.667 \times 10^{29}$	$1.783 \times 10^{-5}$	$3.816 \times 10^{-9}$
327	$1.12 \times 10^{-29}$	$1.786 \times 10^{29}$	$1.940 \times 10^{-5}$	$4.517 \times 10^{-9}$
337	$1.09 \times 10^{-29}$	$1.835 \times 10^{29}$	$2.001 \times 10^{-5}$	$4.807 \times 10^{-9}$
360	$1.02 \times 10^{-29}$	$1.961 \times 10^{29}$	$2.150 \times 10^{-5}$	$5.548 \times 10^{-9}$
393	$9.35 \times 10^{-30}$	$2.139 \times 10^{29}$	$2.345 \times 10^{-5}$	$6.597 \times 10^{-9}$

### Conclusions

In the present work the authors propose a simple model to estimate the bulk deformation of static pulsar in a binary system due to the companion compact NS. The model is applied to the system PSR B1913+16, whose parameters are given in table 1, It shows a consistent behavior with the similar classical predictions from Newtonian potential as the eccentricity is diminished. The model predicts the bulk modulus  $BM$  of the infinite nuclear matter and estimates the amount of the deformation. . The model produces the axial elongation of the observed NS as well as the eccentricity. The eccentricity is found to be of order  $10^{-5}$  and the total elongation is of the order  $10^{-9} km$ . The proposed model requires modification to be effectively used in pulsar data analyses. The authors will introduce the modified model in next publication.

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طريقة لتقدير التشكل الميكانيكي لنابض راديوي موجود ضمن النظام النجمي الثنائي ؛ نبض- راد- نظام بي للاحداثيات 1913+16.

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تم اعلان اكتشاف النابض الراديوي (نبض- راد-نظام بي للاحداثيات 1913+16) والمحتوى ضمن نظام نجمي ثنائي في ورقة بحثية لعالمين هما هولز وتايلور عام 1974 وقد نجحنا في تحليل بيانات الأرصاد وتحديد الكميات الحاكمة للنظام على اعتبار أن الحركة تماثل نظام كبلر، مما انتج دوافع كثيرة لقياسات فيزيائية أخرى لخصائص المادة النووية المكونة للنجوم النيترونية. وقمنا في هذا البحث بتقديم نموذج أولي لتقدير التشكل الحادث في مادة أي من نجمين نيترونيين يدوران حول مركز كتلة مشترك نتيجة قوى الجاذبية المتبادلة وتبعاً لقانون نيوتن للجذب لعام. بواسطة هذا النموذج استطعنا تقدير معيار التشكل الحجمي لمادة نووية لامتناهية مكونة لنابض الراديوي، كما أمكن تحديد كميات الاجهاد والانفعال للمادة النووية عند سطح نجم النيتروني يأخذ شكل اهليلج دوراني. وأظهرت الحسابات أن النموذج الأولي متوافق من حيث حساب مركبات القوى والجهد عند نقاط متعددة في جسم النجم النيتروني وذلك بعد أخذ النهايات حتى رتبة مربع معامل الاختلاف، مع النموذج الأبسط والذي يتكون من نجمين كرويين. وبتطبيق النموذج علي حالة النابض الراديوي (نبض- راد- نظام بي للاحداثيات 1913+16) أظهرت الحسابات أن معامل اختلاف الشكل يتراوح حول الرتبة  $10^{-5}$  وأن الاستطالة على طول المحور- زد، وهي استطالة محورية، تكون من الرتبة  $10^{-9}$  كيلو متر.