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The Hadron Resonance Gas Model with The Effect of Magnetic Field

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THIS work addresses the thermodynamics of quantum chromodynamics (QCD) matter. We study in detail the thermodynamics utilizing the hadron resonance gas model (HRG). The response of QCD matter to the magnetic field is additionally examined. The thermodynamics such as (pressure, number density, energy density, entropy, and magnetization) are determined taking the main concepts of the extensive statistics at zero and non-zero magnetic field, . The magnetic field affects the free energy by dividing it into a vacuum and thermal contributions. The vacuum contribution is represented by setting (), while the thermal term at (). The extensive thermodynamics can emerge from the resonance hadron gas model which is incorporated in the present work. The results are obtained using QCD particle Data Group up to 10 GeV and confronted to the lattice results. Moreover, the effect of the magnetic field on QCD matter proved that QCD matter is a paramagnetic matter which exhibits apositive magnetization.

Keywords: Extensive statistics, HRG, QCD matter, Zero and non-zero magnetic field.

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Introduction

One of the most intriguing questions that scientists are trying to answer is “How was the universe created?” The answer might start with the Big Bang theory, which states that the universe was born as a result of a gigantic explosion that created a fireball. The fireball was made of dense, hot quarks and plasma gluons (QGP), then the fireball began to expand and cool and the elementary particles began to coalesce. The formation of particles in their hadronic phases begins during and after freezing of QGP [1].

Several efforts have been made to study the phase transition between hadronic and QGP theoretically and experimentally. From the collision of heavy ions or two accelerating hadrons to extremely high energies such as the proton-proton (pp), the (Pb-Pb) collision at the Large Hadron Collider (LHC), and (Au -Au) collision at the Relativistic Heavy Ion Collider (RHIC) [2]. QGP matter can exist for a very short time, then it begins to expand leading to recombination of quarks and gluons forming the hadronic matter again at a critical temperature [3, 4, 5].

The hadrons are interacting strongly and can be described by the quantum chromodynamic (QCD) [6, 7]. Additionally, the properties of the hot hadronic matter are best described as the properties of a statistical system, at which we can extract the thermodynamical quantities of such system depending on the Hadron Resonance Gas Model (HRG) [8]. HRG can be studied at finite and vainshing chemical potential[9, 10, 11]. QCD can be described non-perturbatively in which the coupling constant approaches the unity [12, 13], this non-perturbative method is called lattice QCD (LQCD) [14, 15, 16, 17, 18, 19]

HRG is a powerful tool for describing the thermal properties of QCD matter and reproducing LQCD results at low temperatures [20]. However, it does not match LQCD at high temperature. This may be the result of neglected interactions

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between particles, which are very effective at high temperatures [21, 22]. In addition, the HRG model has been studied in a series of papers [23, 24, 25, 26, 27] to study the effects of magnetic fields on QCD matter. This is an excellent review of the effects of magnetism. Magnetic field in vacuum [28] and magnetic field in lattice QCD such as [29, 30, 31]. The external magnetic field B is generated by spectator particles (particles do not contribute to the interaction) in non-central heavy ion collisions. The non-central heavy ion collision diagram allows us to estimate the induced magnetic field [32, 33].Numerical estimation of the magnetic field Au-Au in (RHIC) at Which produces a magnetic field Gauss and for Pb-Pb in (LHC) at Gauss [34, 35, 36].

The effects of magnetic catalyst (MC) and / or inverse magnetic catalyst (IMC) mechanisms still lack a complete understanding of the thermodynamics of quantum chromodynamics (QCD). This is the main motivation for this study to further investigate non-extensive statistics as well as extensive statistics. Strong magnetic fields play important roles in physical systems such as early cosmic explanations, phase transitions, and cosmology [37]. In addition, the magnetic field generated by peripheral heavy ion collisions in a relativistic heavy ion collider (RHIC) or the Large Hadron Collider (LHC) [38, 33]. Analysis of particle formation at different energies has been studied in [39, 40], with extensiveand non-extensiveapplication to black holes where black holes have emerged as a extensive system [41].

Recently, and up to three decades the non-extensive statistics have been applied to many physical systems and showed great success in the description of QCD matter [42], neutron stars [43], and cosmology [44]. The work is organized as follows: section2 represents the hadron resonance gas model and the thermodynamical quantities of the hadronic system at zero and non-zero magnetic fields. SubSection 2.1, discussed the vacuum and thermal free energy contributions. The results and discussion is represented in secion3. Finally, the concluding remarks are introduced, followed by appendix.

Hadron Resonance Gas Model (HRG)

The focus of this work is to examine the extensive thermodynamics in the hadronic system. Beginning with the extensive thermodynamics which can be emerged within the HRG model in which the latter describes the hadron gas as a statistical system with some thermal parameters such as the temperature and the chemical potential. The foremost known form of the entropy is the Boltzmann-Gibbs (BG) for various and discrete states W, as follows,

$$S_{BG} = -k_B \sum_{i=1}^{W} P_i \; ln\, P_i, \tag{1}$$

with k_B is the Boltzmann constant, and the sum ∑_(i=1)^W▒ P_i=1 takes into account all the possible microscopic configurations W, and the probability of each state i is p_i. For the particular case $P_i=1/W$ and for equal probabilities Eq. (1) can be rewritten [45, 46],

$$S_{BG}=K_B \;\; ln \;\; W. \tag{2}$$

In the thermal equilibrium, the probability of the system is defined in terms of the temperature, T

$$P_i=e^(-\beta E_i)/Z\_BG , \tag{3}$$

where E_i is the energy of the state of the system, β=1/(k_B T) and Z_{BG} is the partition function, the latter is defined as [47],

$$Z_{BG}=\sum\_iW \;\; e\text{-}\beta E_i . \tag{4}$$

The canonical partition function has been studied by different methods, e.g. Refs.[48, 49, 50, 51, 52]. The most remarkable point in Boltzmann-Gibbs statistics is the additive property, where a system composed of two subsystems A_1 and A_2 so that the total entropy can be written as,

$$S_{BG}(A)=S_{BG}(A_1)+S_{BG}(A_2). \tag{5}$$

In the grand-canonical ensemble, the partition function of an ideal gas consisting of hadrons and resonances for the i particle is given as [53]

$$\ln Z_i = \pm \frac{V g_i}{(2\pi)^3} \int d^3p \ln\left[1 \pm \lambda_i \exp\left(\frac{-E_i(p)}{T}\right)\right], \tag{6}$$

with the grand canonical partition function is just a sum over all resonances $(ln\;\; Z=\sum_i ln\;\; Z_i)$ where ± refer to fermions and bosons, respectively, is degeneracy, λ_i is defined as [54]

$$\lambda_i(\mu,T) = exp\left(\frac{\mu_s S_i + \mu_B B_i + \mu_q Q_i}{T}\right), \tag{7}$$

where the chemical potential is defined μ_s, μ_B, μ_q are the strange, baryon and quark chemical potential, respectively, multiplied by corresponding quantum numbers S, B, Q, is the relativistic particle energy of mass m_i, in which ($\hbar$=c=k_B=1).

In the present work, we are going to study thermodynamics at zero and non-zero magnetic fields in both statistics. Now We represent the free energy formula followed by the free energy with the effect of the magnetic field to derive the thermodynamics of the hadronic system. First, the free energy reads [55]

$$F=F_{vac}+F_{therm} \quad (8)$$

where and are vacuum and thermal energies respectively. The free energy F is given in terms of the total internal energy U [1],

$$F(V,T)=U-T \quad S \quad (9)$$

Then replacing all the quantities by their corresponding densities, (*e.g.* $s= S/V, \varepsilon=U/V, f=F/V=P$ - The general form would be the Gibbs-Duham relation which is given by eq. (10) [1] in the presence of the magnetic field,

$$\varepsilon = Ts + Bm_B - p \quad (10)$$

where ($m_B=M_B/V$), , and the magnetization is defined as [56]

$$q \quad M_B=(-\partial F/\partial B). \quad (11)$$

Once the partition function is known, all thermodynamical observables can be calculated e.g. the pressure, P, number density, n, energy density, ε , and entropy for particle **i** [1]. Moreover, the free energy is directly related to the partition function so that the thermodynamical quantities can be got from the free energy too as $F_{thermal}(V,T)=-\beta^{-1} \ln Z(V,\beta)$:

$$p = \left(\frac{-\partial F}{\partial V}\right), \quad N = \left(\frac{-\partial F}{\partial \mu}\right), \quad S = \left(-\frac{\partial F}{\partial T}\right) \quad (12)$$

It is known that the partition function *In Z* and *F* are extensive quantities which leads to that, the derivative with respect to the V converts to (1/V) this satisfied at the V thermodynamic limit (i.e at V⟶ ∞).).

The free energy density contributions

In this part, we introduce the free energy different contributions according to Eq. (8) at zero and non-zero magnetic fields, in other words for the neutral and charged particles respectively.

• Firstly, Free energy density at zero magnetic field [55, 57].

$$f(s) = \mp \sum_i \sum_{s_z} \int \frac{d^3p}{(2\pi)^3}\left(\frac{E_i(p)}{2} + T\log\left[1 \pm exp\left(\frac{\mu_i - E_i(p)}{T}\right)\right]\right); q_i = 0, q_i \neq 0 \quad (13)$$

Again, the relativistic energy for particle i in three-dimension momentum P ,

$$E_i(p) = \sqrt{p^2 + m_i^2} \quad (14)$$

Where E_p is the relativistic energy for neutral and charged particles.

• Free energy density at non-zero magnetic field B≠0 [55, 58].

$$f_{charge}(s) = \mp \sum_i \sum_{s_z} \sum_{k=0}^{\infty} \frac{|q_i|B}{(2\pi)^2} \int dp_z \left(\frac{E_i(p_z,k,s_z)}{2} + T\log\left[1 \pm \exp\left(\frac{\mu_i - E_i(p_z,k,s_z)}{T}\right)\right]\right); q_i \neq 0 \quad (15)$$

where the integral over d^3p in eq. (6) is polarized in z-direction for i^{th} particle,

$$\int d^3 p \rightarrow 2\pi |Q_i| eB_z \sum_k \sum_{sz} \int dp_z \quad (16)$$

In comparison with eq. (8), the first part is the contribution due to vacuum and the second part due to the thermal contribution. Where ∓ corresponds to bosons (lower sign) and to fermions (upper sign) which in turn due to the spin sectors bosons with integer spin and for fermions is half-integer. The spin takes the values, (s_z=-s,..,s) is the z-component of the particle spin (s). Where the modified energy due to Landau levels, k, is defined as [59],

$$E_{i_z}(p_z,k,s_z) = \sqrt{p_z^2 + m_i^2 + 2|q_i|\,B\,\left(k - s_z + \frac{1}{2}\right)} \quad (17)$$

With $q_i = Q_i \; e$, is the charge of the particle i, mass m_i of charger particle i, and the electron charge e.[1]. In the present work, we performed the numerical calculations for Particle Data Group (PDG) with masses up to 10 GeV [60].

It is found that the vacuum parts in both eqs. (15, 13) are ultraviolet divergent and need to be normalized through dimensional regularization method [61], and we followed the steps the details in [55] to get the following and to perform the integration in the mentioned equations see Appendix 5

As mentioned above, free energy is composed of two parts, the vacuum part, and the thermal part. We are going to define each part at zero and non-zero magnetic fields. Firstly, at $B = 0$ the vacuum part is defined as,

1In eqs. (15, 13) we add another summation over all the hadronic particles i, and also we can add the chemical potential in the exponential of the thermal part

$$f_{vac}(s,B=0)=\pm(2s+1)\frac{(q\ B)^2}{8\pi^2}v^2\left[\frac{1}{\epsilon}+\frac{3}{4}-\frac{\gamma}{2}-\frac{1}{2}\log\left(\frac{2q\ B}{4\pi a^2}\right)-\frac{1}{2}\log(v)\right] \quad (18)$$

with parameter ϵ and scale and Euler Lagrange γ, where v=m^2/(2q B) removes the dependence on B itself.

And the normalized free energy vacuum part at B=0 is defined as,

$$f_{vac}(s,B\neq 0)=\mp\frac{(q\ B)^2}{8\pi^2}\sum_{\alpha}\left[\left(-\frac{\epsilon}{2}+\gamma+\log\left(\frac{4\pi a^2}{2q\ B}\right)-1\right)\left(\frac{1}{12}-\frac{(x+\alpha)^2}{2}+\frac{x+\alpha}{2}\right)-\zeta'(-1,x+\alpha)\right] \quad (19)$$

where ζ is the Hurwitz function (see Appendix 5) emerged from the conversion of the sum over k, and .The change in the free energy density due to the magnetic field, one subtract the B = 0 eq.(18). For all details of the normalization see refs.[55, 62], we write down the final normalized free energies as obtained in [55]. In the present work we have performed the calculation for different sectors of the spin of particles (s 0, 1/2 , and 1). Consequently, the renormalized free energy for those sectors of spins is defined as:

$$\Delta f^{vac}(0)=\frac{(q\ B)^2}{8\pi^2}\left[\zeta'\left(-1,v+\frac{1}{2}\right)+\frac{v^2}{4}-\frac{v^2}{2}\ \log(v)+\frac{\log(v)+1}{24}\right] \quad (20)$$

$$\Delta f^{vac}(1/2)=-\frac{(q\ B)^2}{4\pi^2}\left[\zeta'(-1,v)+\frac{v}{2}\log(v)+\frac{v^2}{4}-\frac{v^2}{2}\log(v)-\frac{\log(v)+1}{12}\right] \quad (21)$$

$$\Delta f_{vac}(1)=\frac{3(q\ B)^2}{8\pi^2}\left[\zeta'\left(-1,v-\frac{1}{2}\right)+\frac{3}{2}\left(v+\frac{1}{2}\right)\log\left(v+\frac{1}{2}\right)+\frac{3}{2}\left(v-\frac{1}{2}\right)\right.$$

$$\left.\log\left(v-\frac{1}{2}\right)+\frac{v^2}{2}\left(\frac{1}{2}-\log(v)\right)-7\,\frac{\log(v)+1}{24}\right] \quad (22)$$

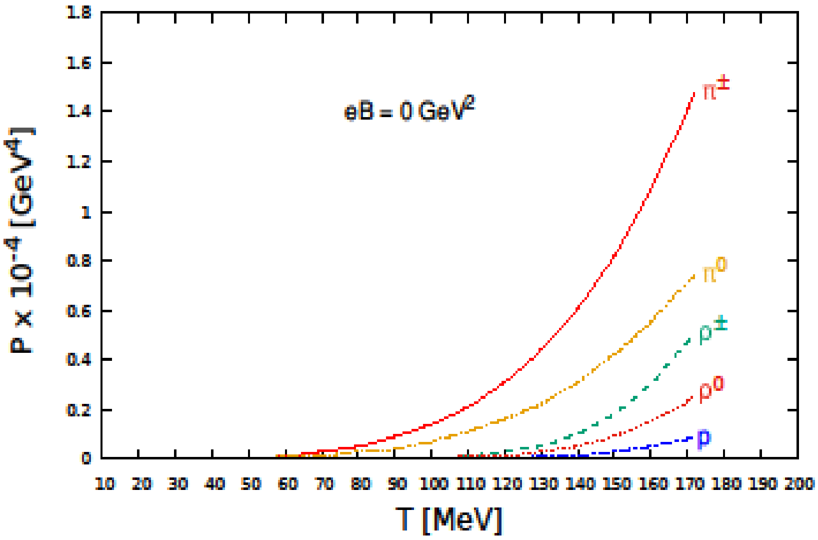
Results and Discussion

We present the results of extensive thermodynamic statistics. The impact of the magnetic field is studied in which the thermodynamical observables such as,(pressure, energy density, entropy, and magnetization) are investigated. These quantities are studied at zero and non-zero magnetic fields. The response of the QCD matter to the magnetic field is examined through the magnetization which exhibits positive values which indicates that QCD is a paramagnetic material. All our results are studied at vanishing baryon chemical potential.

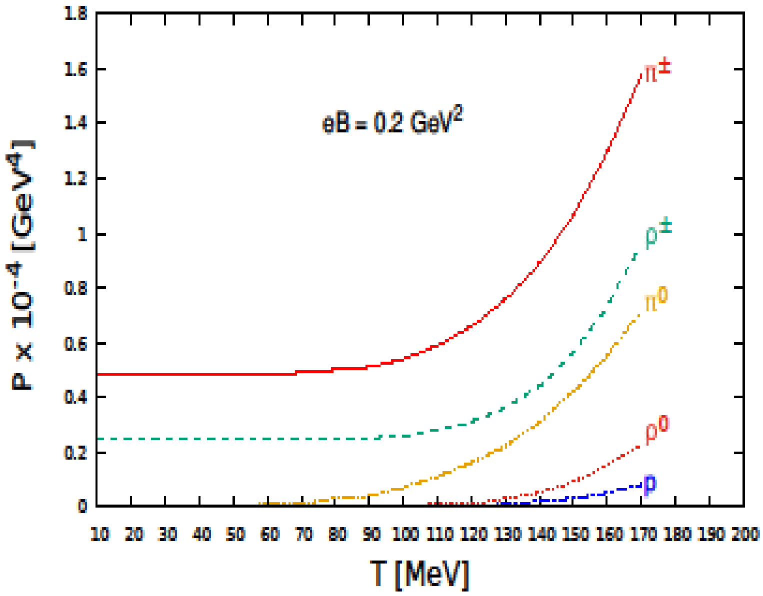
Extensive Individual Particle Contribution and Thermodynamics

Figure 1 shows the pressure for the particles p,$\rho^{0,\pm}$,$\pi^{0,\pm}$ the left panel (0) for vanishing magnetic field and the right panel (0) for magnetic field (eB = 0.2 GeV^2) versus the temperature T. It is noticed that the charged particles pressure differs from zero magnetic fields to the one at a finite magnetic field. This difference appears at temperature (100 < T < 140 MeV), in which ρ^± shifted by , and shifted by ~0.2×10^{-4} GeV^4. π^± shifted by ~0.5×10^{-4} *GeV4*. The domination of the charged pions clearly appears in both cases. It is noticeable that the pressure for the charged particles is shifted because of the vacuum term included and confirms the effect of the magnetic field only on the charged hadrons. Figure 2 represents the EoS of QCD matter at (eB=0,0.2,0.3 *GeV*^{2}). The pressure (Fig.1) versus the temperature shows a reasonable agreement with the lattice results [30], the pressure also exhibits an increase by increasing the magnetic field.

Figure 2 represents the energy density, one can see the increase in the energy density with increasing the magnetic field. The entropy and magnetization of the QCD matter (Fig. 3) are calculated at (*eB=0.2,0.3 GeV*^{2}), it is remarkable that the magnetization increases with increasing the magnetic field as appeared in (2), while the increase in the entropy is slight as shown in Fig. 2. The slightly quantitative increase in the entropy at zero magnetic field than that at non-zero magnetic field is due to the full scan of the particle number of charged and neutral particles at *eB=0*, while at eB≠0 is restricted by the charged particles only and for some sectors of particle spin. This can be shown clearly in the other thermodynamics such as the pressure and the energy density in which we restricted both cases of zero and nonzero magnetic field for some particle sectors. So that, the cases at *eB=0* are lower than others.



(a)The (colored online) pressure for individual particles at vanishing magnetic field



(b) The (colored online) pressure for individual particles at non-zero magnetic field.

Fig. 1. The pressure for particles at zero and non-zero magnetic field calculated for p (blue), (dotted red and green), (yellow and solid red) in left and right panels.

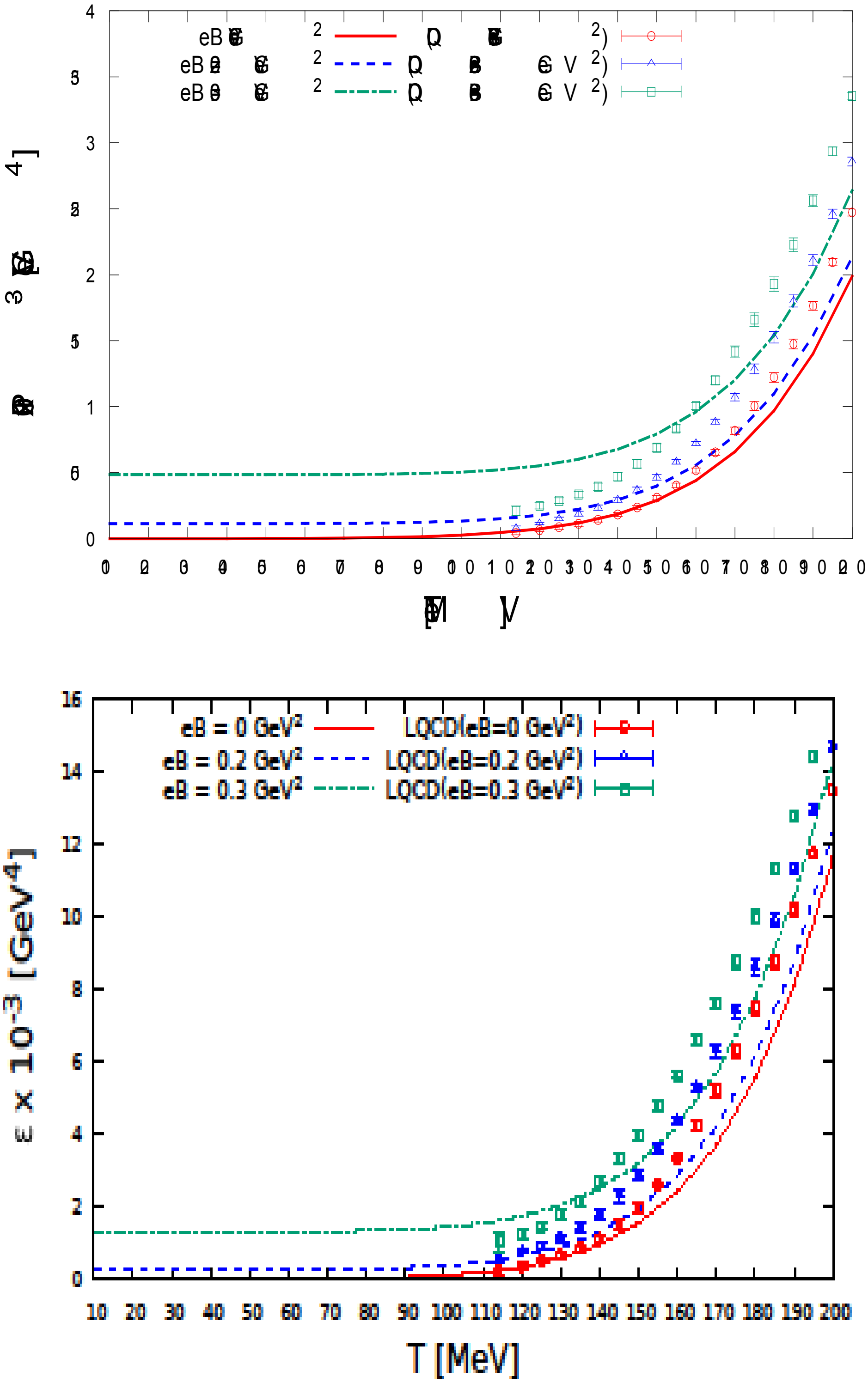
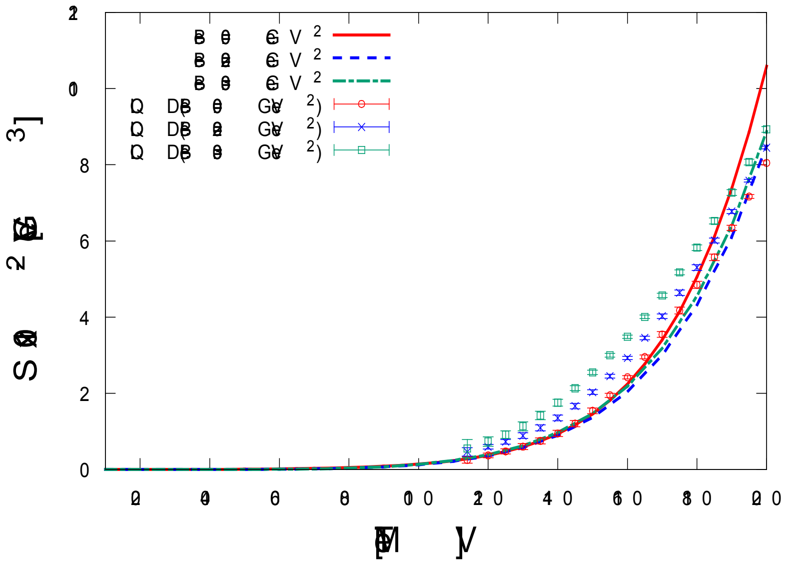


Fig. 2. The Colored-online pressure (upper panel) versus the temperature, with (red-line),(dashed-blue) and (dotted-dashed green) at (eB=0,0.2,0.3 GeV2) respectively compared with the corresponding lattice data (colored-online symbols). The energy density (lower-panel) at the same conditions.



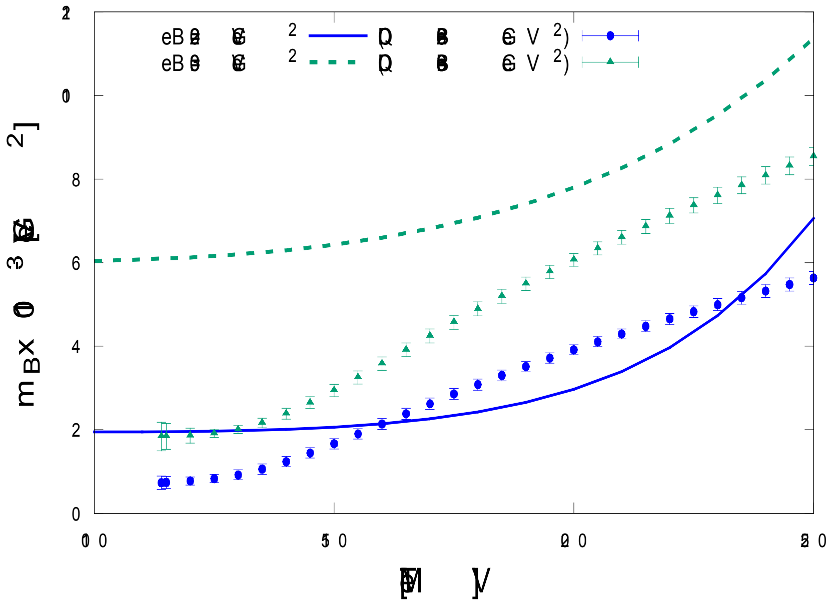


Fig. 3. The Colored-online entropy (upper panel) versus the temperature with (red-line),(dashed-blue) and (dotted-dashed green) at at (eB=0,0.2,0.3 GeV2) respectively compared with the corresponding lattice data (colored-online symbols), and finally the magnetization (lower-panel).

TABLE 1. Sample list of hadrons in our PDG [60].

MC-Nr (Particle code)	Name	Mass [GeV]	Width	dg	nb	ns	nc	nb	I3	Q	Decay Chnnl	MC-Nr	daughters	BrDaughters-MC-Nrs
22	Gamma	0	0	2	0	0	0	0	1	0	1	1	1	22
211	Pion-(+)	0.13957	0	1	0	0	0	0	3	1	1	1	1	211
111	Pion-(0)	0.13498	0	1	0	0	0	0	3	0	1	1	1	111
-211	Pion-(-)	0.13957	0	1	0	0	0	0	3	-1	1	1	1	-211

Conclusion

In the present work, we have explored the effect of the magnetic field on extensive thermodynamics. The free energy is separated into two parts, the vacuum, and thermal contributions. We have applied the magnetic field on both terms in which the vacuum-free energy needs to be regularized according to the reduction in the dimension from 3 to 1. Accordingly, the relativistic energy of the charged particles has been modified with the Landau levels and polarized spin in the z-direction. The response of the QCD matter to the magnetic field is examined through the magnetization via the temperature and various magnetic fields. In conclusion, the magnetization slightly increases at low temperature, however it increases dramatically and rapidly with increasing the magnetic field and the temperature which indicates that, QCD is a paramagnetic matter. However the magnetization shows a disagreement with the lattice results. All results are confronted to the available lattice results at zero and non-zero magnetic field which show a reasonable agreement with the extensive thermodynamics.

Appendix A

These are the sample of the tables that we have used in our calculations, and includes up to mass . Table 1 contains the mass code, mass in GeV, width, degeneracy, baryon, strangeness, charmness, bottomness quantum numbers, a third component of isospin, charge, and the number of decay channels respectively. Table 1 also includes information for the daughter in case of the decay of the same particle with “Br” the branching ratio and “Nrs” the number of resonances. In the present work, we did not include the decay channels. In addition, the degeneracy appears in table (1), then the spin can be extracted from the relation between the degeneracy and spin eq.(23),

$$g_i=2si+1 \tag{23}$$

• *Some useful definitions*

The vacuum part can be solved by using the standard integration in dimensions (Ref. [63])

$$\int_{-\infty}^{\infty} \frac{d^d p}{(2\pi)^d}\sqrt{p^2+M^2} = \frac{1}{(4\pi)^{d/2}}\frac{\Gamma(-1/2-d/2)}{\Gamma(-1/2)}\left(M^2\right)^{1/2+d/2}, \tag{24}$$

with $\Gamma(-1/2)=-2\sqrt{\pi}$.

• The Hurwitz function is defined as

$$\sum_{k=0}^{\infty} \frac{1}{(v+k)^z} = \zeta(z,v), \tag{25}$$

with the expansion and asymptotic behavior [64]

$$\zeta(-1+\epsilon/2,v) \approx -\frac{1}{12}-\frac{v^2}{2}+\frac{v}{2}+\frac{\epsilon}{2}\zeta'(-1,v)+O(\epsilon^2), \tag{26}$$

$$\zeta'(-1,v) = \frac{1}{12}-\frac{v^2}{4}+\left(\frac{1}{12}-\frac{v}{2}+\frac{v^2}{2}\right)\log(v)+O(v^{-2}). \tag{27}$$

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